Temporal and spatial variation of fundamental constants: theory and observations

V.V. Flambaum
School of Physics, The University of New South Wales, Sydney NSW 2052, Australia

Space-time variation of the fundamental constants is suggested by theories unifying gravity with other interactions. It also can explain fine tuning of the fundamental constants which is needed for life to appear. Review of recent works devoted to the variation of the fine structure constant α, strong interaction and fundamental masses (Higgs vacuum) is presented. The results from Big Bang nucleosynthesis, quasar absorption spectra, and Oklo natural nuclear reactor data give us the space-time variation on the Universe lifetime scale. Comparison of different atomic clocks gives us the present time variation. Assuming linear variation with time we can compare different results. The best limit on the variation of the electron-to-proton mass ratio $\mu = m_e/M_p$ and $X_e = m_e/\Lambda_{QCD}$ follows from the quasar absorption spectra [1]: $\dot{\mu}/\mu = \dot{X}_e/X_e = (1 \pm 3) \times 10^{-16}$ yr^{-1}. A combination of this result and the atomic clock results [2, 3] gives the best limt on variation of α: $\dot{\alpha}/\alpha = (-0.8 \pm 0.8) \times 10^{-16}$ yr^{-1}. The Oklo natural reactor gives the best limit on the variation of $X_s = m_s/\Lambda_{QCD}$ where m_s is the strange quark mass [4, 5]: $|\dot{X}_s/X_s| < 10^{-18}$ yr^{-1}. Note that the Oklo data can not give us any limit on the variation of α since the effect of α there is much smaller than the effect of X_s and should be neglected.

Huge enhancement of the relative variation effects happens in transitions between close atomic, molecular and nuclear energy levels. We suggest several new cases where the levels are very narrow. Large enhancement of the variation effects is also possible in cold atomic and molecular collisions near Feshbach resonance.

How changing physical constants and violation of local position invariance may occur? Light scalar fields very naturally appear in modern cosmological models, affecting parameters of the Standard Model (e.g. α). Cosmological variations of these scalar fields should occur because of drastic changes of matter composition in Universe: the latest such event is rather recent (about 5 billion years ago), from matter to dark energy domination. Massive bodies (stars or galaxies) can also affect physical constants. They have large scalar charge S proportional to number of particles which produces a Coulomb-like scalar field $U = S/r$. This leads to a variation of the fundamental constants proportional to the gravitational potential, e.g. $\delta\alpha/\alpha = k_\alpha \delta(GM/rc^2)$. We compare different manifestations of this effect. The strongest limits [6] $k_\alpha + 0.17k_e = (-3.5 \pm 6) \times 10^{-7}$ and $k_\alpha + 0.13k_q = (-1 \pm 17) \times 10^{-7}$ are obtained from the measurements of dependence of atomic frequencies on the distance from Sun [2, 7] (the distance varies due to the ellipticity of the Earth's orbit).

I. INTRODUCTION

A search for the variations of the fundamental constants is currently a very popular research topic. There are several reasons to search for the variation of the fundamental "constants" of nature in space and time. Fisrt, the Universe is evolving. Several phase transitions happened at the early Universe accompanied by dramatic changes in vacuum energy, fundamental masses, fundamental interactions (electromagnetic, weak and strong) and properties of elementary particle (e.g. confinement of quarks). At later stages the equation of state of the Universe continued to evolve, from the radiation domination (pressure $p = \epsilon/3$ where ϵ is the energy density) to cold matter domination ($p \ll \epsilon$) and "recently", about 5 billion years ago, to dark energy domination ($p \approx -\epsilon$). In view of these dramatic changes it seems natural to check if there is any evolution in the values of the fundamental constants during this process.

Another reason: the spatial variation can explain fine tuning of the fundamental constants which allows humans (and any life) to appear. Indeed, it is well known that if the values of some fundamental constants (e.g. related to the strong interaction) would be even by 1% different we could not appear. If we assume that the fundamental constants vary in space, this problem of fine tuning may be resolved in a most natural way. We appeared in the area of the Universe where the values of the fundamental constants are consistent with our existence.

Another argument comes from theories unifying gravity and other interactions. Some theories suggest the possibility of spatial and temporal variation of physical "constants" in the Universe (see, e.g. [8, 9]). Moreover, there exists a mechanism for making all coupling constants and masses of elementary particles both space and time dependent, and influenced by local circumstances (see e.g. [9–12]). The variation of coupling constants can be non-monotonic (for example, damped oscillations).

These variations are usually associated with the effect of massless (or very light) scalar fields. One candidate is the dilaton: a scalar which appears in string theories together with a graviton, in a massless multiplet of closed string excitations. Other scalars naturally appear in cosmological models, in which our Universe is a "brane" floating in a space of larger dimensions. The scalars are simply brane coordinates in extra dimensions. However, the only relevant scalar field recently discovered, the cosmological dark energy, so far does not show visible variations. Available observational limits on physical constant variations at present time are quite strict, allowing only scalar coupling tiny in comparison with gravity.

A possible explanation was suggested by Damour et al [10, 11] who pointed out that cosmological evolution of scalars naturally leads to their self-decoupling. Damour and Polyakov have further suggested that variations should happen when the scalars get excited by some physical change in the Universe, such as the phase transitions or other drastic change in the equation of State of the Universe. They considered few of them, but since the time of their paper a new fascinating transition has been discovered: from matter dominated (decelerating) era to dark energy dominated (accelerating) era. It is relatively recent event, corresponding to cosmological redshift $z \approx 0.5$.

The time dependence of the perturbation related to it can be calculated, and it turned out [13, 14] that the self-decoupling process is effective enough to explain why after this transition the variation of constants is as small as observed in laboratory experiments at the present time, as well as at Oklo ($\sim$ 2 billion years ago or $z = 0.14$) and isotopes ratios in meteorites (4.6 billion years to now, $z = 0.45 - 0$), while being at the same time consistent with possible observations of the variations of the electromagnetic fine structure constant at $z \sim 1$.

Another topic we will address here is similar variations of constants in space, near massive bodies such as stars (Sun), pulsars, Galaxy. We will compare possible sensitivities related with different possible objects, point out limitations following from some recent experiments with atomic clocks and suggest new measurements (this part is based on Ref. [6]).

Recent observations have produced several hints for the variation of the fine structure constant, $\alpha = e^2/\hbar c$, strength constant of the strong interaction and masses in Big Bang nucleosynthesis, quasar absorption spectra and Oklo natural nuclear reactor data (see e.g.[15–17, 19]) . However, a majority of publications report only limits on possible variations (see e.g. reviews [9, 20]). A very sensitive method to study the variation in a laboratory consists of the comparison of different optical and microwave atomic clocks (see recent measurements in [2, 21–28]).

Sensitivity to temporal variation of the fundamental constants may be strongly enhanced in transitions between narrow close levels of different nature. Huge enhancement of the relative variation effects can be obtained in transition between the almost degenerate levels in atoms [29–33], molecules [1, 34–37] and nuclei [38, 39].

II. OPTICAL SPECTRA

A. Comparison of quasar absorption spectra with laboratory spectra

To perform measurements of α variation by comparison of cosmic and laboratory optical spectra we developed a new approach [29, 40] which improves the sensitivity to a variation of α by more than an order of magnitude. The relative value of any relativistic corrections to atomic transition frequencies is proportional to α^2. These corrections can exceed the fine structure interval between the excited levels by an order of magnitude (for example, an s-wave electron does not have the spin-orbit splitting but it has the maximal relativistic correction to energy). The relativistic corrections vary very strongly from atom to atom and can have opposite signs in different transitions (for example, in s-p and d-p transitions). Thus, any variation of α could be revealed by comparing different transitions in different atoms in cosmic and laboratory spectra.

This method provides an order of magnitude precision gain compared to measurements of the fine structure interval. Relativistic many-body calculations are used to reveal the dependence of atomic frequencies on α for a range of atomic species observed in quasar absorption spectra [29, 30, 40, 41]. It is convenient to present results for the transition frequencies as functions of α^2 in the form

$$\omega = \omega_0 + qx, \tag{1}$$

where $x = (\frac{\alpha}{\alpha_0})^2 - 1 \approx \frac{2\delta\alpha}{\alpha}$ and ω_0 is a laboratory frequency of a particular transition. We stress that the second term contributes only if α deviates from the laboratory value α_0. We performed accurate many-body calculations of the coefficients q for all transtions of astrophysical interest (strong E1 transtions from the ground state) in Mg, Mg II, Fe II, Cr II, Ni II, Al II, Al III, Si II, and Zn II. It is very important that this set of transtions contains three large classes : positive shifters (large positive coefficients $q > 1000$ cm^{-1}), negative shifters (large negative coefficients $q < -1000$ cm^{-1}) and anchor lines with small values of q. This gives us an excellent control of systematic errors since systematic effects do not "know" about sign and magnitude of q. Comparison of cosmic frequencies ω and laboratory frequencies ω_0 allows us to measure $\frac{\delta\alpha}{\alpha}$.

Three independent samples of data contaning 143 absorption systems spread over red shift range $0.2 < z < 4.2$. The fit of the data gives [15] is $\frac{\delta\alpha}{\alpha} = (-0.543 \pm 0.116) \times 10^{-5}$. If one assumes the linear dependence of α on time, the fit of the data gives $d\ln\alpha/dt = (6.40 \pm 1.35) \times 10^{-16}$ per year (over time interval about 12 billion years). A very extensive search for possible systematic errors has shown that known systematic effects can not explain the result (It is still not completely excluded that the effect may be imitated by a large change of abundances of isotopes during last 10 billion years. We have checked that different isotopic abundances for any single element can not imitate the observed effect. It may be an improbable "conspiracy" of several elements).

Recently our method and calculations [29, 30, 40, 41] were used by two other groups [42, 43]. However, they have not detected any variation of α. Most probably, the difference is explained by some undiscovered systematic effects. However, another explanation is not excluded. These results of [15] are based on the data from the Keck telescope which is located in the Northen hemisphere (Hawaii). The results of [42, 43] are based on the data from the different telescope (VLT) located in the Southern hemisphere (Chile). Therefore, the difference in the results may be explained by the spatial variation of α.

Recently the results of [42] were questioned in Ref. [44]. Re-analysis of Ref. [42] data revealed flawed parameter estimation methods. The authors of [44] claim that the same spectral data fitted more accurately give $\frac{\delta\alpha}{\alpha} = (-0.44 \pm 0.16) \times 10^{-5}$ (instead of $\frac{\delta\alpha}{\alpha} = (-0.06 \pm 0.06) \times 10^{-5}$ in Ref.[42]). However, even this revised result may require further revision.

Using opportunity I would like to ask for new, more accurate laboratory measurements of UV transition frequencies which have been observed in the quasar absorption spectra. The "shopping list" is presented in [45]. We also need the laboratory measurements of isotopic shifts - see [45]. We have performed very complicated calculations of these isotopic shifts [46]. However, the accuracy of these calculations in atoms and ions with open d-shell (like Fe II, Ni II, Cr II, Mn II, Ti II) may be very low. The measurements for at list few lines are needed to test these calculations. These measurements would be very important for a study of evolution of isotope abundances in the Universe, to exclude the systematic effects in the search for α variation and to test models of nuclear reactions in stars and supernovi.

B. Optical clocks

Optical clocks also include transitions which have positive, negative or small constributions of the relativistic corrections to frequencies. We used the same methods of the relativistic many-body calculations to calculate the dependence on α [29, 30, 47]. The coefficients q for optical clock transitions may be substantially larger than in cosmic transitions since the clock transitions are often in heavy atoms (Hg II, Yb II, Yb III, etc.) while cosmic spectra contain mostly light atoms lines ($Z < 33$). The relativistic effects are proporitional to $Z^2\alpha^2$.

III. ENHANCED EFFECTS OF α VARIATION IN ATOMS

An enhancement of the relative effect of α variation can be obtained in transition between the almost degenerate levels in Dy atom [29, 30]. These levels move in opposite directions if α varies. The relative variation may be presented as $\delta\omega/\omega = K\delta\alpha/\alpha$ where the coefficient K exceeds 10^8. Specific values of $K = 2q/\omega$ are different for different hyperfine components and isotopes which have different ω; $q = 30,000$ cm^{-1}, $\omega \sim 10^{-4}$ cm^{-1}. An experiment is currently underway to place limits on α variation using this transition [32, 33]. The current limit is $\dot{\alpha}/\alpha = (-2.7 \pm 2.6) \times 10^{-15}$ yr^{-1}. Unfortunately, one of the levels has quite a large linewidth and this limits the accuracy.

Several enhanced effects of α variation in atoms have been calculated in [31].

IV. ENHANCED EFFECTS OF α VARIATION IN MOLECULES

The relative effect of α variation in microwave transitions between very close and narrow rotational-hyperfine levels may be enhanced 2-3 orders of magnitude in diatomic molecules with unpaired electrons like LaS, LaO, LuS, LuO, YbF and similar molecular ions [35]. The enhancement is a result of cancellation between the hyperfine and rotational intervals; $\delta\omega/\omega = K\delta\alpha/\alpha$ where the coefficients K are between 10 and 1000.

This enhancement may also exist in a large number of molecules due to cancelation between the ground state fine structure ω_f and vibrational interval ω_v ($\omega = \omega_f - n\omega_v \approx 0$, $\delta\omega/\omega = K(2\delta\alpha/\alpha - 0.5\delta\mu/\mu)$, $K \gg 1$, $\mu = m_e/M_p$ - see [37]). The intervals between the levels are conveniently located in microwave frequency range and the level widths are very small. Required accuracy of the shift measurements is about 0.01-1 Hz. As examples, we consider molecules Cl_2^+, CuS, IrC, SiBr and HfF^+. An enhancement due to the cancellation between the electron and vibrational intervals in Cs_2 molecule was suggested earlier by D. DeMille [34].

V. VARIATION OF THE STRONG INTERACTION

The hypothetical unification of all interactions implies that a variation in α should be accompanied by a variation of the strong interaction strength and the fundamental masses. For example, the grand unification models discussed in Ref. [8] predicts the quantum chromodynamics (QCD) scale Λ_{QCD} (defined as the position of the Landau pole in the logarithm for the running strong coupling constant, $\alpha_s(r) \sim 1/\ln(\Lambda_{QCD} r/\hbar c)$) is modified as $\delta\Lambda_{QCD}/\Lambda_{QCD} \approx 34\ \delta\alpha/\alpha$. The variations of quark mass m_q and electron masses m_e (related to variation of the Higgs vaccuum field which generates fundamental masses) in this model are given by $\delta m/m \sim 70\ \delta\alpha/\alpha$, giving an estimate of the variation for the dimensionless ratio

$$\frac{\delta(m/\Lambda_{QCD})}{(m/\Lambda_{QCD})} \sim 35\frac{\delta\alpha}{\alpha} \qquad (2)$$

The coefficient here is model dependent but large values are generic for grand unification models in which modifications come from high energy scales; they appear because the running strong-coupling constant and Higgs constants (related to mass) run faster than α.

Indeed, the strong (i=3), and electroweak (i=1,2) inverse coupling constants have the following dependence on the scale ν and normalization point ν_0:

$$\alpha_i^{-1}(\nu) = \alpha_i^{-1}(\nu_0) + b_i ln(\nu/\nu_0) \tag{3}$$

In the Standard Model $2\pi b_i = 41/10, -19/6, -7$ and the couplings are related as $\alpha^{-1} = (5/3)\alpha_1^{-1} + \alpha_2^{-1}$. There are two popular scenarios of Grand Unification: with the standard model as well as for its minimal supersymmetric extension (MSSM). In the latter case 3 curves for α_i (i=1,2,3) cross at one point, believed to be a "root" of the three branches (electromagnetic, weak and strong). One may select the unification point for ν_0, and for example, $\nu = m_Z$ is the Z-boson mass (String theories lead to more complicated "trees", which however also have a singly "root", at a string scale Λ_s and bare string coupling g_s.)

Basically there are two possibilities. If one assumes that only $\alpha_{GUT} \equiv \alpha_i(\nu_0)$ varies, the eqn (3) gives us the same shifts for all inverse couplings

$$\delta\alpha_1^{-1} = \delta\alpha_2^{-1} = \delta\alpha_3^{-1} = \delta\alpha_{GUT}^{-1} \tag{4}$$

If so, the variation of the strong interaction constant $\alpha_3(m_z)$ is much larger than the variation of the em constant α, $\delta\alpha_3/\alpha_3 = (\alpha_3/\alpha_1)\delta\alpha_1/\alpha_1$.

Another option is the variation of the GUT scale (ν/ν_0 in eqn (3)). If so, quite different relations between variations of the three coupling follows

$$\delta\alpha_1^{-1}/b_1 = \delta\alpha_2^{-1}/b_2 = \delta\alpha_3^{-1}/b_3 \tag{5}$$

Note that now variations have different sign since the one loop coefficients b_i have different sign for 1 and 2,3. Another unclear issue is the modification of lepton/quark masses, which are proportional to Higgs vacuum expectation value and thus depend on the mechanism of electroweak symmetry breaking.

If these models are correct, the variation in electron or quark masses and the strong interaction scale may be easier to detect than a variation in α. One can only measure the variation of dimensionless quantities. The variation of m_q/Λ_{QCD} can be extracted from consideration of Big Band nucleosynthesis, quasar absorption spectra and the Oklo natural nuclear reactor, which was active about 1.8 billion years ago [4]. There are some hints for the variation in Big Bang Nucleosynthesis ($\sim 10^{-3}$ - see Ref.[16]) and Oklo ($\sim 10^{-9}$ - see Ref.[17]) data. However, these results are not confirmed by new studies [5, 18].

The results from Oklo natural nuclear reactor are based on the measurement of the position of very low energy resonance ($E_r = 0.1$ eV) in neutron capture by ^{149}Sm nucleus. The estimate of the shift of this resonance induced by the variation of α have been done long time ago in works [48]. Recently we performed a rough estimate of the effect of the variation of m_q/Λ_{QCD} [4]. The final result is

$$\delta E_r \approx 10^6 eV(\frac{\delta\alpha}{\alpha} - 10\frac{\delta X_q}{X_q} + 100\frac{\delta X_s}{X_s}) \tag{6}$$

where $X_q = m_q/\Lambda_{QCD}$, $X_s = m_s/\Lambda_{QCD}$, $m_q = (m_u + m_d)/2$ and m_s is the strange quark mass. Refs. [5] found that $|\delta E_r| < 0.1$ eV. This gives us a limit

$$|0.01\frac{\delta\alpha}{\alpha} - 0.1\frac{\delta X_q}{X_q} + \frac{\delta X_s}{X_s}| < 10^{-9} \tag{7}$$

The contribution of the α variation in this equation is very small and should be neglected since the accuracy of the calculation of the main term is low. Thus, the Oklo data can not give any limit on the variation of α. Assuming linear time dependence during last 2 billion years we obtain an estimate $|\dot{X}_s/X_s| < 10^{-18}$ yr^{-1}.

The proton mass is proportional to Λ_{QCD} ($M_p \sim 3\Lambda_{QCD}$), therefore, the measurements of the variation of the electron-to-proton mass ratio $\mu = m_e/M_p$ is equivalent to the measurements of the variation of $X_e = m_e/\Lambda_{QCD}$. Two new results have been obtained recently using quasar absorption spectra. In our paper [49] the varition of the ratio of the hydrogen hyperfine frequency to optical frequencies in ions have been measured. The result is consistent with no variation of $X_e = m_e/\Lambda_{QCD}$. However, in the recent paper [19] the variation was detected at the level of 4 standard deviations: $\frac{\delta X_e}{X_e} = \frac{\delta\mu}{\mu} = (-2.4 \pm 0.6) \times 10^{-5}$. This result is based on the hydrogen molecule spectra. Note, however, that the difference between the zero result of [49] and non-zero result of [19] may be explained by a space-time variation of X_e. The variation of X_e in [19] is substantially larger than the variation of α measured in [15, 42]. This may be considered as an argument in favour of Grand Unification theories of the variation [8].

Recently we obtained the limit on the space-time variation of the ratio of the proton mass to the electron mass based on comparison of quasar absorption spectra of NH_3 with CO, HCO^+ and HCN rotational spectra [1]. For the inversion transition in NH_3 ($\lambda \approx 1.25$ cm^{-1}) the relative frequency shift is significantly enhanced: $\delta\omega/\omega = 4.46\,\delta\mu/\mu$. This enhancement allows one to increase sensitivity to the variation of μ using NH_3 spectra for high redshift objects. We use published data on microwave spectra of the object B0218+357 to place the limit $\delta\mu/\mu = (-0.6 \pm 1.9) \times 10^{-6}$ at redshift $z = 0.6847$; this limit is several times better than the limits obtained by different methods and may be significantly improved. Assuming linear time dependence we obtain [1] $\dot{\mu}/\mu = \dot{X}_e/X_e = (1 \pm 3) \times 10^{-16}$ yr^{-1}.

VI. MICROWAVE CLOCKS

Karshenboim [50] has pointed out that measurements of ratios of hyperfine structure intervals in different atoms are sensitive to variations in nuclear magnetic moments. However, the magnetic moments are not the fundamental parameters and can not be directly compared with any theory of the variations. Atomic and nuclear calculations are needed for the interpretation of the measurements. We have performed both atomic calculations of α dependence [29, 30, 47] and nuclear calculations of $X_q = m_q/\Lambda_{QCD}$ dependence [3] for all microwave transitions of current experimental interest including hyperfine transitions in ^{133}Cs, ^{87}Rb, ^{171}Yb$^+$, ^{199}Hg$^+$, ^{111}Cd, ^{129}Xe, ^{139}La, ^{1}H, ^{2}H and ^{3}He. The results for the dependence of the transition frequencies on variation of α, $X_e = m_e/\Lambda_{QCD}$ and $X_q = m_q/\Lambda_{QCD}$ are presented in Ref.[3] (see the final results in the Table IV of Ref.[3]). Also, one can find there experimental limits on these variations which follow from the recent measurements. The accuracy is approaching 10^{-15} per year. This may be compared to the sensitivity $\sim 10^{-5} - 10^{-6}$ per 10^{10} years obtained using the quasar absorption spectra.

According to Ref. [3] the frequency ratio Y of the 282-nm ^{199}Hg$^+$ optical clock transition to the ground state hyperfine transition in ^{133}Cs has the following dependence on the fundamental constants:

$$\dot{Y}/Y = -6\dot{\alpha}/\alpha - \dot{\mu}/\mu - 0.01\dot{X}_q/X_q \tag{8}$$

In the work [2] this ratio has been measured: $\dot{Y}/Y = (0.37 \pm 0.39) \times 10^{-15}$ yr^{-1}. Assuming linear time dependence we obtained the quasar result [1] $\dot{\mu}/\mu = \dot{X}_e/X_e = (1 \pm 3) \times 10^{-16}$ yr^{-1}. A combination of this result and the atomic clock result [2] for Y gives the best limt on the variation of α: $\dot{\alpha}/\alpha = (-0.8 \pm 0.8) \times 10^{-16}$ yr^{-1}. Here we neglected the small ($\sim 1\%$) contribution of X_q.

VII. ENHANCED EFFECT OF VARIATION OF α AND STRONG INTERACTION IN UV TRANSITION OF ^{229}TH NUCLEUS (NUCLEAR CLOCK)

A very narrow level (3.5 ± 1) eV above the ground state exists in ^{229}Th nucleus [51] (in [52] the energy is (5.5 ± 1) eV, in [53] the energy is (7.6 ± 0.5) eV). The position of this level was determined from the energy differences of many high-energy γ-transitions (between 25 and 320 KeV) to the ground and excited states. The subtraction produces the large uncertainty in the position of the 3.5 eV excited state. The width of this level is estimated to be about 10^{-4} Hz [54]. This would explain why it is so hard to find the direct radiation in this very weak transition. The direct measurements have only given experimental limits on the width and energy of this transition (see e.g. [55]). A detailed discussion of the measurements (including several unconfirmed claims of the detection of the direct radiation) is presented in Ref.[54]. However, the search for the direct radiation continues [56].

The ^{229}Th transition is very narrow and can be investigated with laser spectroscopy. This makes ^{229}Th a possible reference for an optical clock of very high accuracy, and opens a new possibility for a laboratory search for the varitation of the fundamental constants [39].

As it is shown in Ref. [38] there is an additional very important advantage. The relative effects of variation of α and m_q/Λ_{QCD} are enhanced by 5 orders of magnitude. A rough estimate for the relative variation of the ^{229}Th transition frequency is

$$\frac{\delta\omega}{\omega} \approx 10^5 (2\frac{\delta\alpha}{\alpha} + 0.5\frac{\delta X_q}{X_q} - 5\frac{\delta X_s}{X_s})\frac{7\, eV}{\omega} \tag{9}$$

where $X_q = m_q/\Lambda_{QCD}$, $X_s = m_s/\Lambda_{QCD}$, $m_q = (m_u + m_d)/2$ and m_s is the strange quark mass. Therefore, the Th experiment would have the potential of improving the sensitivity to temporal variation of the fundamental constants by many orders of magnitude.

Note that there are other narrow low-energy levels in nuclei, e.g. 76 eV level in ^{235}U with the 26.6 minutes lifetime (see e.g.[39]). One may expect a similar enhancement there. Unfortunetely, this level can not be reached with usual lasers. In principle, it may be investigated using a free-electron laser or synchrotron radiation. However, the accuracy of the frequency measurements is much lower in this case.

VIII. ENHANCEMENT OF VARIATION OF FUNDAMENTAL CONSTANTS IN ULTRACOLD ATOM AND MOLECULE SYSTEMS NEAR FESHBACH RESONANCES

Scattering length A, which can be measured in Bose-Einstein condensate and Feshbach molecule experiments, is extremely sensitive to the variation of the electron-to-proton mass ratio $\mu = m_e/m_p$ or $X_e = m_e/\Lambda_{QCD}$ [57].

$$\frac{\delta A}{A} = K\frac{\delta\mu}{\mu} = K\frac{\delta X_e}{X_e}, \tag{10}$$

where K is the enhancement factor. For example, for Cs-Cs collisions we obtained $K \sim 400$. With the Feshbach resonance, however, one is given the flexibility to adjust position of the resonance using external fields. Near a narrow magnetic or an optical Feshbach resonance the enhancement factor K may be increased by many orders of magnitude.

IX. CHANGING PHYSICS NEAR MASSIVE BODIES

In this section I follow Ref. [6].

The reason gravity is so important at large scales is that its effect is additive. The same should be true for massless (or very light) scalars: its effect near large body is proportional to the number of particles in it.

For not-too-relativistic objects, like the usual stars or planets, both their total mass M and the total scalar charge Q are simply proportional to the number of nucleons in them, and thus the scalar field is simply proportional to the gravitational potential

$$\phi - \phi_0 = \kappa(GM/rc^2)\,. \tag{11}$$

Therefore, we expect that the fundamental constants would also depend on the position via the gravitational potential at the the measurement point.

Naively, one may think that the larger is the dimensionless gravity potential (GM/rc^2) of the object considered, the better. However, different objects allow for quite different accuracy.

Let us mention few possibilities, using as a comparison parameter the product of gravity potential divided by the tentative relative accuracy

$$P = (GM/rc^2)/(accuracy) \tag{12}$$

(i) Gravity potential on Earth is changing due to ellipticity of its orbit: the corresponding variation of the Sun graviational potential is $\delta(GM/rc^2) = 3.3 \cdot 10^{-10}$. The accuracy of atomic clocks in laboratory conditions approaches 10^{-16}, and so $P \sim 3 \cdot 10^6$. However, comparing clocks on Earth and distant satellite one may get variation of the Earth graviational potential $\delta(GM/rc^2) \sim 10^{-9}$ and $P \sim 10^7$. The space mission was recently discussed, e.g. in the proposal [59] and references therein. Note that the matter composition of Earth and Sun is very different, therefore, the proportionality coefficients κ in Eq (11) may also be different. Indeed, the first example (Sun) is mainly sensitive to the scalar potentials of electrons and protons while the second example (Earth) is in addition sensitive to the scalar potentials of neutrons and virtual mesons mediating the nuclear forces (the nuclear binding energy).

(ii) Sun (or other ordinary stars) has $GM/rc^2 \sim 2 \cdot 10^{-7}$. Assuming accuracy 10^{-8} in the measurements of atomic spectra near the surface we get $P \sim 10$. However, a mission with modern atomic clocks sent to the Sun would have $P \sim 10^8$ or so, see details in the proposal [58].

(iii) The stars at different positions inside our (or other) Galaxy have gravitational potential difference of the order of 10^{-7}, and (like for the Sun edge) one would expect $P \sim 10$. Clouds which give the observable absorption lines in quasar spectra have also different gravitational potentials (relative to Earth), of comparable magnitude.

(iv) White/brown dwarfs have $GM/rc^2 \sim 3 \cdot 10^{-4}$, and in some cases rather low temperature. We thus get $P \sim 3 \cdot 10^4$.

(v) Neutron stars have very large gravitational potential $GM/rc^2 \sim 0.1$, but high temperature and magnetic fields make accuracy of atomic spectroscopy rather problematic, we give tentative accuracy 1 percent. $P \sim 10$.

(vi) Black holes, in spite of its large gravitational potential, have no scalar field outside the Shwartzschield radius, and thus are not useful for our purpose.

Accuracy of the atomic clocks is so high because they use extremely narrow lines. At this stage, therefore, star spectroscopy seem not to be competitive: the situation may change if narrow lines be identified.

Now let us see what is the best limit available today. As an example we consider recent work [2] who obtained the following value for the half-year variation of the frequency ratio of two atomic clocks: (i) optical transitions in mercury ions $^{199}Hg^+$ and (ii) hyperfine splitting in ^{133}Cs (the frequency standard). The limit obtained is

$$\delta ln(\frac{\omega_{Hg}}{\omega_{Cs}}) = (0.7 \pm 1.2) \cdot 10^{-15} \tag{13}$$

For Cs/Hg frequency ratio of these clocks the dependence on the fundamental constants was evaluated in [3] with the result

$$\delta ln(\frac{\omega_{Hg}}{\omega_{Cs}}) = -6\frac{\delta\alpha}{\alpha} - 0.01\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} - \frac{\delta(m_e/M_p)}{(m_e/M_p)} \tag{14}$$

Another work [60] compare H and ^{133}Cs hyperfine transitions. The amplitude of the half-year variation found were

$$|\delta ln(\omega_H/\omega_{Cs})| < 7 \cdot 10^{-15} \tag{15}$$

The sensitivity [3]

$$\delta ln(\frac{\omega_H}{\omega_{Cs}}) = -0.83\frac{\delta\alpha}{\alpha} - 0.11\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} \tag{16}$$

There is no sensitivity to m_e/M_p because they are both hyperfine transitions.

As motivated above, we assume that scalar and gravitational potentials are proportional to each other, and thus introduce parameters k_i as follows

$$\frac{\delta\alpha}{\alpha} = k_\alpha \delta(\frac{GM}{rc^2}) \tag{17}$$

$$\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = k_q \delta(\frac{GM}{rc^2}) \tag{18}$$

$$\frac{\delta(m_e/\Lambda_{QCD})}{(m_e/\Lambda_{QCD})} = \frac{\delta(m_e/M_p)}{(m_e/M_p)} = k_e \delta(\frac{GM}{rc^2}) \tag{19}$$

where in the r.h.s. stands half-year variation of Sun's gravitational potential on Earth.

In such terms, the results of Cs/Hg frequency ratio measurement [2] can be rewritten as

$$k_\alpha + 0.17k_e = (-3.5 \pm 6) \cdot 10^{-7} \tag{20}$$

The results of Cs/H frequency ratio measurement [60] can be presented as

$$|k_\alpha + 0.13k_q| < 2.5 \cdot 10^{-5} \tag{21}$$

Finally, the result of recent measurement [7] of Cs/H frequency ratio can be presented as

$$k_\alpha + 0.13k_q = (-1 \pm 17) \cdot 10^{-7} \tag{22}$$

The sensitivity coefficients for other clocks have been discussed above.

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